

The Core Radius of a Star Cluster Containing a Massive Black Hole

Douglas C. HEGGIE

*School of Mathematics and Maxwell Institute for Mathematical Sciences, University of Edinburgh,
 King's Buildings, Edinburgh EH9 3JZ, UK
 d.c.heggie@ed.ac.uk*

Piet HUT

*Institute for Advanced Study, Princeton, NJ 08540, USA
 piet@ias.edu*

Shin MINESHIGE

*Yukawa Institute for Theoretical Physics, Kyoto University, Sakyo-ku, Kyoto 606-8502, Japan
 minesige@yukawa.kyoto-u.ac.jp*

Jun MAKINO

*Center for Computational Astrophysics, National Astronomical Observatory of Japan,
 2-21-1 Ohsawa, Mitaka, Tokyo 181-8588, Japan
 makino@th.nao.ac.jp*

and

Holger BAUMGARDT

*Argelander Institute for Astronomy (AIfA), Auf dem Hügel 71, 53121 Bonn, Germany
 holger@astro.uni-bonn.de*

(Received 2000 December 31; accepted 2001 January 1)

Abstract

We present a theoretical framework which establishes how the core radius of a star cluster varies with the mass of an assumed central black hole. Our result is that $r_c/r_h \propto (M_{bh}/M)^{3/4}$. The theory compares favourably with a number of simulations of this problem, which extend to black hole masses of order 10% of the cluster mass. Though strictly limited as yet to clusters with stars of equal mass, our conclusion strengthens the view that clusters with large core radii are the most promising candidates in which to find a massive black hole.

Key words: Galaxy: globular clusters: general – black hole physics – stellar dynamics

1. Introduction

Though the existence of intermediate-mass black holes in star clusters remains controversial, our theoretical understanding of the problem has advanced on two fronts. First, important conclusions have been reached from purely theoretical considerations, including the density profile of the cusp surrounding the black hole (Bahcall & Wolf, 1976; Shapiro & Lightman, 1976). This can be understood as the response of the stellar distribution to the steady transport of stars into the vicinity of the black hole, where they are tidally disrupted, contributing to the growth of the black hole. Two-body relaxation is the vital process which controls the rate at which this flux can be sustained. Second, a succession of simulations have added much detail to the general theoretical picture. These simulations have been based on a variety of techniques: Monte Carlo methods based on a Fokker-Planck treatment of relaxation with an anisotropic distribution of velocities (Shapiro & Marchant, 1978; Duncan & Shapiro, 1982; Freitag & Benz, 2002); finite-difference solution of the Fokker-Planck equation for both anisotropic (Cohn & Kulsrud, 1978) and isotropic distribution functions (Murphy et al., 1991); gas models, in which relax-

ation is mimicked by a suitably crafted form for thermal conductivity in a self-gravitating gas (Amaro-Seoane et al., 2004); a tree code (Arabadjis, 1997); and, most recently, direct N -body simulations (Baumgardt et al., 2004a,b, 2005).

In many of these studies, emphasis is given to the details of the cusp and the growth of the black hole, and less attention is paid to the evolution of the star cluster. In this letter we shall show that rather simple considerations allow us to predict also the evolution of the structure of the cluster, in particular its core radius. The basic idea is a familiar one in the stellar dynamics of star clusters, where it was introduced by Hénon (1975). This idea is that the flux of energy from the centre of a star cluster must reach an equilibrium with the flux (by relaxation) across the outer parts of the cluster, conventionally taken to be the half-mass radius. If too much energy is generated the core of the star cluster must expand to quench the generation of energy there, no matter what is the mechanism of energy generation; while insufficient generation of energy leads to the familiar process of core collapse. For the case of a central black hole, energy is generated as stars fall by relaxation towards the radius around the black hole at

which they are disrupted¹. Though the link between the energy generated in the cusp and the expansion of the system has been discussed previously (see Sec.5), we believe that the theoretical estimate of the core radius is novel.

Here is an outline of the letter. In the following section we review the basic results on the flow of energy through the cusp around the black hole. Then we apply Hénon's argument, which establishes the way in which the radius of the core varies with the mass of the black hole. Next we compare our prediction with existing data from simulations, and finally discuss the place of our result within the literature on this subject.

2. The Energy Flux from a Central Black Hole

We consider a cluster of mass M containing a central black hole of mass M_{bh} . The black hole is surrounded by a cusp, which merges at its edge into a core of nearly constant density (Fig.1). Beyond the core radius the density falls off again in the halo of the star cluster.

Inside the cusp, the flux of energy at radius r is

$$\mathcal{E} \sim \frac{\rho r^3 v^2}{t_r}.$$

where ρ is the stellar density, v^2 is the mean square stellar velocity, and t_r is the relaxation time. This is given by $t_r \sim \frac{v^3}{G^2 m \rho \ln \Lambda}$, where $\ln \Lambda$ is the Coulomb logarithm and m is the individual stellar mass. Hence

$$\mathcal{E} \sim \frac{G^2 m \rho^2 r^3 \ln \Lambda}{v}.$$

In the part of the cluster where the potential is dominated by the black hole, i.e. $r < \frac{GM_{bh}}{v_c^2}$, where v_c is the velocity

dispersion in the core of the cluster, we have $v^2 \sim \frac{GM_{bh}}{r}$, and so

$$\mathcal{E} \sim \frac{G^{3/2} m \rho^2 r^{7/2} \ln \Lambda}{M_{bh}^{1/2}}.$$

If the flux is independent of r (Lightman & Shapiro, 1977) we get $\rho \propto r^{-7/4}$.

At the edge of the cusp we have $v^2 \sim v_c^2$, and so the radius of the cusp is

$$r_{cusp} \sim \frac{GM_{bh}}{v_c^2}. \quad (1)$$

At this radius we also have $\rho \sim \rho_c$, where ρ_c is the density in the core, and so

$$\mathcal{E} \sim \frac{G^5 m \rho_c^2 M_{bh}^3 \ln \Lambda}{v_c^7}. \quad (2)$$

¹ Our arguments also apply, in principle, to a purely classical, point-mass idealisation, in which stars simply accumulate around the black hole at small distances.

3. The Radius of the Core

In steady post-collapse expansion this must balance the energy flux at the half-mass radius, which is

$$\mathcal{E}_h \sim \frac{M v_h^2}{t_{rh}} \quad (3)$$

$$\sim \frac{G^2 m M \rho_h \ln \Lambda}{v_h}, \quad (4)$$

where the subscript h denotes conditions at the half-mass radius. Estimating $\rho_h \sim \frac{M}{r_h^3}$ and equating \mathcal{E}_h to \mathcal{E} , we obtain

$$\frac{M^2}{v_h r_h^3} \sim \frac{G^3 \rho_c^2 M_{bh}^3}{v_c^7}.$$

If conditions are approximately isothermal between the core radius, r_c , and the half-mass radius, we can estimate

$$\rho_c \sim \frac{M}{r_h^3} \frac{r_h^2}{r_c^2} \text{ and } v_c^2 \sim v_h^2 \sim \frac{GM}{r_h}, \text{ whence}$$

$$\frac{r_c}{r_h} \sim \left(\frac{M_{bh}}{M} \right)^{3/4}. \quad (5)$$

This scaling of core radius with black hole mass is expected to be approximately valid provided that the resulting core radius exceeds that of the cusp around the black hole (eq.1). Using the above estimate for v_c^2 , it follows that

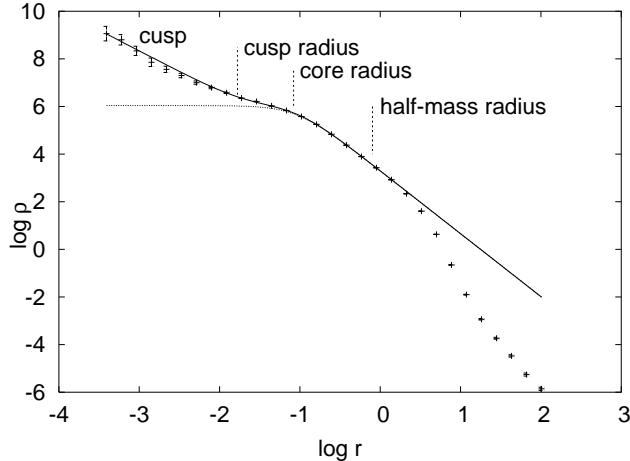
$$\frac{r_{cusp}}{r_c} \sim \left(\frac{M_{bh}}{M} \right)^{1/4}. \quad (6)$$

For a sufficiently massive black hole, the density profile may show no sign of a core radius. This conclusion was already reached by Marchant & Shapiro (1979), though they modelled systems with a core radius independent of the black hole mass. A fit to numerical data (Sec.4), however, yields a coefficient of about 0.7 in eq.(6), and we find that, even up to a black hole mass of order 10% of the cluster mass, the radius of the cusp is less than half the core radius.

4. Comparison with Simulations

Baumgardt et al. (2004a) have carried out a series of N -body simulations with stars of equal mass and several different values for the initial black hole mass. In addition, unpublished data from some of these runs allow us to measure the core radius for intermediate values of the black hole mass, which increases during the course of each simulation. The measurement of core radius is not entirely straightforward, however. While the N -body code itself gives a current value of the ‘‘core radius’’ (Aarseth, 2003), this is based on a density-weighted average of the distances of stars from the density centre. Because the computed density around the black hole increases with increasing black hole mass, this approach introduces a mass-dependent bias which leads to an underestimate of the true core radius for large black hole masses. Baumgardt

Fig. 1. Example of a fit of eq.(7) to the density profile. Data outside $r = 2$ were not included in the fit, which is for the case $M_{bh}/M = 0.0075$. The dotted line shows the profile without the contribution from the cusp, and in this figure the “cusp radius” is defined to be the radius at which the contributions from the core and cusp are equal.



et al. (2005) adopted a more observational procedure, and estimated the core radius as the radius at which the surface density drops to half its central value; but this is also biased in the same way. Therefore we have taken a different approach, in which we fit the density distribution in the simulation by a template which describes the cusp around the black hole embedded in a core. Specifically, we fitted the density at radius r by

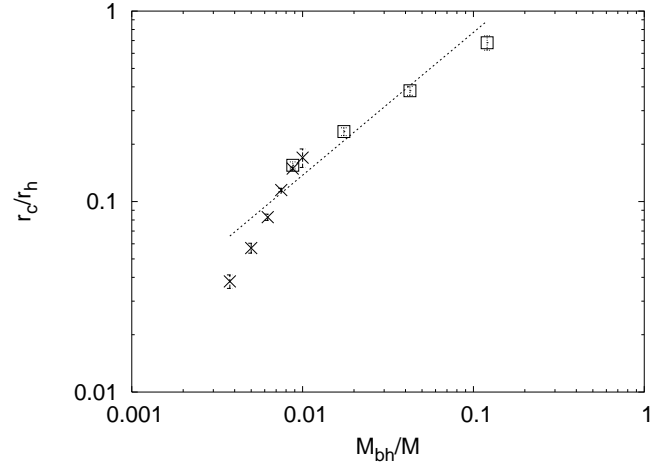
$$\rho(r) = (ar^{-7/4} + b)(1 + r^2/r_c^2)^d, \quad (7)$$

where a, b, r_c, d are parameters. For $a = 0$ (no cusp) this resembles closely the so-called EFF model for the distribution of density in a star cluster with a core (Elson et al., 1987; Mackey & Gilmore, 2003). The fit is not good at large radii; by experimenting, we found that the mean square residual was minimised if data outside about $2r_h$ were rejected (Fig.1).

Figure 2 shows fits of our theory to the data we have obtained in this way. The points on the left (crosses) come from one simulation, and plot the evolution of r_c/r_h as the black hole grows. The early points in particular do not fit the predicted power law very well, and we consider that these correspond to the period during which equilibrium has not yet been established between the flux of energy at the half-mass radius and that provided by the cusp. The points on the right (boxes) represent four different simulations, with different initial black hole mass, late in the evolution. The agreement with our theory is better, but the theory depends implicitly on a homology assumption, and the slightly discrepant slope may be due to small departures from homology; this is not unexpected, given that the values of r_c/r_h extend up to about 0.8.

The important result to be obtained from this figure is that the core radius *increases* with black hole mass, which is the opposite of the conclusion that would be drawn

Fig. 2. Dependence of core radius (in units of the half-mass radius) on black hole mass (in units of the total cluster mass). The line is the graph of $r_c/r_h = 4.3(M_{bh}/M)^{3/4}$ (cf. eq.5), which is the best fit. Boxes: data from the four profiles in Baumgardt et al. (2004a); crosses: data at various times during the run with $M_{bh}(0) = 266M_\odot$. The error bars are the standard error provided by the fitting package, and include the contribution of sampling errors in the individual density measurements.



from Table 1 of Baumgardt et al. (2005)². There the core radius was estimated from the radius at which the surface density drops to half its central value (see above), and the fact that their paper considered clusters with a stellar mass function is not relevant.

5. Discussion

In this Letter we have argued that there is a balance between the energy production of the cusp and the energy required for expansion of the entire cluster. Previously Marchant & Shapiro (1980) considered that there should be a balance between energy production of the cusp and that required for expansion of the *core*. This, however, leads to the relation $r_c/r_{cusp} \sim \text{constant}$, independent of the mass of the black hole and cluster. This is certainly incorrect when the black hole mass is very small. Shapiro (1977), McMillan et al. (1981) and Duncan & Shapiro (1982) also, in effect, equated the generation of energy in the cusp to the energy required to expand the core; in fact, however, most of the energy passes through the core to the half-mass radius, as it provides the energy needed for the expansion of the entire cluster. Baumgardt et al. (2004a) equated the energy generation to that required at r_h , but assumed a constant value of r_h/r_c . Yuan & Zhong (1990) and Dokuchaev (1991) took a homology model for the stellar system, which was thus characterised by a single scale radius.

The theory we have presented applies only to systems with stars of equal mass, though there is no reason to suppose that it does not extend, with suitable modification of

² We also note that the last value of $R_C/R_{h,pro}$ in that Table should be 0.07.

detail, to systems with a realistic mass spectrum. It applies only when sufficient time has elapsed for achieving a balance between the energy produced in the cusp and the flow of energy across the half-mass radius. It does not apply to systems that are so dense (including many galactic nuclei) that the structure of the cusp is dominated by the role of physical collisions, and the power law is altered (Rauch, 1999; Duncan & Shapiro, 1983). Furthermore, in galactic nuclei the relaxation time would usually be too long for the conditions assumed in this Letter to be established.

Acknowledgements

DCH, PH and JM thank SM for his hospitality during a visit to the Yukawa Institute, Kyoto, where much of the work was carried out. This work was supported in part by the Grants-in-Aid of the Ministry of Education, Science, Culture, and Sport (14079205, 16340057 S.M.), and by the Grant-in-Aid for the 21st Century COE “Center for Diversity and Universality in Physics” from the Ministry of Education, Culture, Sports, Science and Technology (MEXT) of Japan.

References

- Aarseth, S. J. 2003, *Gravitational N-Body Simulations*, Cambridge, UK: Cambridge University Press
- Amaro-Seoane, P., Freitag, M., & Spurzem, R. 2004, *MNRAS*, 352, 655
- Arabadjis, J. S. 1997, Ph.D. Thesis, University of Michigan
- Bahcall, J. N., & Wolf, R. A. 1976, *ApJ*, 209, 214
- Baumgardt, H., Makino, J., & Hut, P. 2005, *ApJ*, 620, 238
- Baumgardt, H., Makino, J., & Ebisuzaki, T. 2004a, *ApJ*, 613, 1133
- Baumgardt, H., Makino, J., & Ebisuzaki, T. 2004b, *ApJ*, 613, 1143
- Cohn, H., & Kulsrud, R. M. 1978, *ApJ*, 226, 1087
- Dokuchaev, V. I. 1991, *MNRAS*, 251, 564
- Duncan, M. J., & Shapiro, S. L. 1982, *ApJ*, 253, 921
- Duncan, M. J., & Shapiro, S. L. 1983, *ApJ*, 268, 565
- Elson, R. A. W., Fall, S. M., & Freeman, K. C. 1987, *ApJ*, 323, 54
- Freitag, M., & Benz, W. 2002, *A&A*, 394, 345
- Hénon, M. 1975, *IAU Symp.* 69: Dynamics of Stellar Systems, 69, 133
- Lightman, A. P., & Shapiro, S. L. 1977, *ApJ*, 211, 244
- Mackey, A. D., & Gilmore, G. F. 2003, *MNRAS*, 338, 85
- McMillan, S. L. W., Lightman, A. P., & Cohn, H. 1981, *ApJ*, 251, 436
- Marchant, A. B., & Shapiro, S. L. 1979, *ApJ*, 234, 317
- Marchant, A. B., & Shapiro, S. L. 1980, *ApJ*, 239, 685
- Murphy, B. W., Cohn, H. N., & Durisen, R. H. 1991, *ApJ*, 370, 60
- Rauch, K. P. 1999, *ApJ*, 514, 725
- Shapiro, S. L. 1977, *ApJ*, 217, 281
- Shapiro, S. L., & Lightman, A. P. 1976, *Nature*, 262, 743

- Shapiro, S. L., & Marchant, A. B. 1978, *ApJ*, 225, 603
- Yuan, Z., & Zhong, X. G. 1990, *Ap&SS*, 168, 233